Strategies to Address Mathematics Areas of Concern

MTSS-RtII Mathematics Strategies K-12

Area Of Concern	Strategy	
Computational Fluency	**Board work with class discussion**	A group of students at the board solving a problem provides immediate assessment of skills, as well as a springboard for class discussion. As volunteers at the board complete a problem, other students in the class solve the same problem at their seats. When everyone is finished, the work can be evaluated, and students can ask questions about the work displayed on the board. Conducting a class discussion at this point creates an opportunity to correct common misconceptions and enhance mathematical communication. (See Class discussion.)
	Calculators and computers	Calculators and computers can be used to foster a deeper understanding of mathematical concepts. Along with increasing the speed of numerical computations, these tools allow students to visually explore algebraic and geometric relationships and manipulations. While every effort should be made to ensure that all students achieve fluency with basic arithmetic facts and procedures, calculators allow for the inclusion of students who have not attained mastery in this area. These tools allow students to explore areas of mathematics that might otherwise be inaccessible to them. Calculator and computer applications, games, and activities can also assist students in acquisition of basic skills by providing a nonthreatening means of independent study. Calculators and computers can be used with other forms of technology to integrate mathematics with science and other subject areas. Additionally, they can be used for data collection, simulations, and modeling real-world mathematics.
	Daily performance	Assessing students' daily performance provides teachers with immediate, continual feedback on students' comprehension level. Using student performance and participation in class activities, cooperative groups, and board work, teachers have immediate assessment of the current state of the class, what adjustments need to be made, and how to move forward. Daily performance of students presents many opportunities for informal and ongoing assessment.
	Guided notes	Guided notes provide assessment of student engagement during class time. Students are required to actively respond during instruction, creating an opportunity to assess how accurate and efficient they are with the presented material, as well as how they listen, think, and write in response to the material. Asking students to prepare guided notes for classmates or for a new chapter provides assessment of organizational skills, conceptual understanding, and clarity of thought regarding specific concepts.
	Ongoing assessments	Ongoing assessment provides continual assessment throughout the year of the effectiveness of teaching techniques and materials, and examination of students' individual learning styles for the benefit of teacher and student. The assessment itself should be a teaching tool, allowing students to continue learning through the process and enabling them to learn more about their individual needs. Teachers should be able to gauge students' progress in understanding concepts and applying them; they should be able to envision how to proceed, after evaluating what changes need to be made. Students should be aware of ongoing assessment and assist in planning and constructing some forms of the assessment. Class discussions, directed questions, writing assignments, group assessment, peer evaluation, and daily performance can be used as methods of ongoing assessment.
	Skill reinforcement activities	These activities can be games, competitions, puzzles, and problem-solving activities that reinforce different skills. Making mathematics enjoyable and approachable is vital to keeping students interested and focused; therefore, these activities should be stimulating and motivational. Students should be encouraged to find, explore, and solve real-world problems that interest them. Offering a variety of games and activities can address a wide range of interests, skills, and learning styles. Since successful classroom experiences are great motivators, activities should be offered at various ability levels so that all students can experience success.
	Think/pair/share	Think/pair/share is a discussion strategy where students formulate their own answers to the question posed by the teacher, share their answers with a partner, and listen to their partners' answers without interrupting. This strategy elicits participation from students who are reluctant to speak in front of the entire class.

Area Of Concern	Strategy	
Fact Fluency	**Class discussion**	The teacher or a student can lead class discussions. Discussions should take place regularly and allow students to explain their understanding of a concept or express misconceptions about it. Students should feel comfortable expressing themselves and should be encouraged to question and challenge each other's understanding of mathematical topics. Discussions give teachers a quick assessment of student comprehension and a sense of where the class is as a whole. Learning to express mathematical ideas, questions, and understanding verbally increases retention of material. Class discussions can be used to explore the practical applications of mathematical concepts and connections between these concepts and other problems and disciplines. Like cooperative group activities, class discussions provide an opportunity for students to hear explanations formulated by other students. This is critical because deeper conceptual understanding can come through hearing a concept explained in a variety of different ways. Discussions can also help the teacher and students understand each other's culture and frame of reference.
	Cooperative groups	Cooperative group activities are critical for a cooperative discovery classroom. Students are placed in pairs or groups of four, which can be designed according to ability or arranged heterogeneously. The design should be dependent on the goals of the teacher. Many students will feel more comfortable if they work in a small group prior to participating in whole class activities. Effective group functioning requires that each individual play a defined role in achieving the task. This promotes cooperation and inclusion. In a successful cooperative group environment, each member of the group takes responsibility for completion of the task and each other's learning. Working in cooperative groups helps students to improve their teamwork skills, while also allowing for individual accountability. Since each member may take on more than one role, having a method for assigning roles is helpful, such as drawing out of a hat. Working in groups requires students to be able to verbalize questions and solutions in various ways. Students may need to clarify and organize the material so that others can understand the related concepts and process. **Suggested roles:** **Spokesperson:** organizes and presents the group's project to the rest of the class, answers questions the class may have **Moderator:** keeps the group on task and ensures that each member has an opportunity to contribute ideas **Journalist:** records, organizes, and summarizes information decided upon by the group, designs charts and transparencies for use during oral presentations **Technician:** retrieves and returns equipment and supplies as needed **Administrator:** organizes discussions, allocates work assignments, monitors the safety of investigations, and ensures the security and privacy of the group's work **Manager:** monitors time, alerts the group of elapsed and remaining time to ensure completion of task within the allocated time, checks for participation and completion of designated work
	Oral assessment	Oral assessment is a powerful assessment tool that can be used frequently and in many forms, often allowing the unique opportunity of assessment through interaction. Teachers can collect feedback immediately, assessing conceptual development and students' ability to organize the material. Using methods of assessment that are consistent with methods of instruction can help build students' depth of understanding and enable the teacher to evaluate teaching strategies and techniques. Class discussions, presentations, and projects provide a forum for oral assessment. The provided rubrics can be used for some forms of oral assessment.
	Review games	Games such as Memory™, BINGO™, and simulated quiz shows can be effective motivators when reviewing material studied. For review of vocabulary or skills, Memory™ and BINGO™ are often successful. A quiz show format can be used to incorporate review questions of varying levels of difficulty (recall, analysis, and application).

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<tr><th>Area Of Concern</th><th colspan="2">Strategy</th></tr>
<tr><td rowspan="4">Math Application: Conceptual & Procedural</td><td>Formal presentations</td><td>Formal presentations provide the opportunity to assess students by checking conceptual knowledge, depth of understanding, and ability to present procedures and results in a clear, systematic, succinct, and accurate manner. Welcoming groups from outside the classroom to participate in the assessment can offer new insight for both teacher and students. The rubrics provided can be utilized to assess such presentations.</td></tr>
<tr><td>Individual or group projects</td><td>The classroom is an active, discovery classroom and assessment by projects offers students the opportunity to represent their learning in a similar way. Creativity can be assessed, along with students' ability to take their understanding of concepts to a higher level. Projects not only assess comprehension of new material and previously learned concepts, but also the ability to integrate and apply all of this knowledge. Projects require students to research, analyze, summarize, problem solve, and present material to a variety of audiences. Projects that are presented allow for oral assessment by requiring explanations, defense of ideas, summary of information, application, and answering questions. Teachers and peers can evaluate summation of material, clarity in display and communication, and depth of knowledge in response to questions. Suggested projects can be found throughout the Holt Algebra 2 textbook. Use the provided rubrics as a form of assessment for projects or design specific project rubrics with the class to be used by both teacher and students in the assessment process.</td></tr>
<tr><td>KWL
Know,
Want to know,
Learned</td><td><table><tr><th>K</th><th>W</th><th>L</th></tr><tr><td></td><td></td><td></td></tr></table>
This strategy can be used for class discussions, investigations, and journal writing. Students begin by identifying what they already know (K). Organizing prior knowledge makes it possible to see connections to new concepts. Students then formulate questions for discussion or investigation, focusing on what they want to know (W). Finally, students summarize what they have learned (L). This summary should include answers to earlier questions along with any additional questions that have arisen, prompting more investigations if necessary. For group investigations, a KWL chart may be helpful.</td></tr>
<tr><td>Modeling</td><td>Modeling can be used as a means of formative assessment. In modeling a situation accurately, students will demonstrate an understanding of the underlying mathematical structure. Modeling builds on prior student knowledge and extends it to new areas. Modeling can occur in most areas of mathematics, such as fractions, decimals, ratios, proportions, patterns, transformations, functions, graphs, and data analysis. The teacher plays an important role in monitoring the accuracy of the mathematics involved in modeling a situation.</td></tr>
<tr><td>Problem of the week</td><td>Problems of the week can be used to assess prior knowledge, comprehension of current material, and determining understanding of upcoming concepts. Such problems reference concepts from earlier in the year, previous math courses, or look forward to new concepts. Problems can be designed by the teacher, students, or found on the Internet</td></tr>
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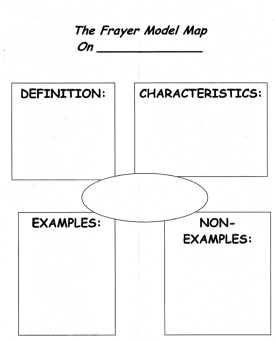
Area of Concern	Strategy	
Algebraic Concepts	**Algebra Tiles**	Algebra tiles are manipulatives the can be used as follows: adding, subtracting, multiplying and dividing signed numbers; modeling simple substitution and the distributive property; solving equations; representing, adding, subtracting, multiplying, dividing and factoring polynomials; and completing the square.
	Cross-Curricular Activity/ Reading	As frequently as possible mathematics should support and connect to subjects currently being taught to students. Readings that support and stimulate interest in the mathematical topic should also be incorporated into lessons. For example, a study of parametric functions might be launched with an article about ships or planes and their respective paths traveled. The Titanic or Andrea Dora shipwrecks are two examples of topics that might be discussed parametrically and factors that could possibly have prevented the tragedies could be explored. Distance and scale can be embedded into geography lesson, chemistry equations can be incorporated in algebra lessons. Flower and leaf growth patterns can bring the study of fractals into the world of science. Oil spills can be integrated into lessons about area and growth rates. The opportunities to integrate mathematics into other subject areas are endless.
	Frayer Model Map	 Use Frayer Model Map to introduce students to new information or to summarize information. To introduce a topic, have students write the word and the definition. Students add the remaining components of the graphic organizer while progressing through the lesson or unit. To summarize topics use at the end of a lesson or unit. Periodically give students a few minutes to discuss their Frayer Model Map entries with a partner at the beginning of a lesson, at the end of a lesson, or while transitioning between activities. Encourage student to enhance and edit their Frayer Model Map entries after each discussion.
	Graphing calculator visualizations	Graphing calculator visualizations spark student interest. They allow students to explore and analyze various relationships that they may not have visualized otherwise. Using this strategy will deepen student understanding and encourage curiosity.
	Hands-on activities	Manipulatives and measurement tools should be used to address various learning styles. They should be used as instructional tools for visualizing concepts and strengthening comprehension. There is great value in learning to use these tools and perform investigations with them. When students are engaged in the learning process by working with manipulatives, the depth of their understanding and retention will be greater. Strategies with manipulatives and procedures for using measurement tools are included in most textbooks.
	Investigation/ Self-discovery	In classrooms focused on discovery learning, various types of investigations should be occurring regularly. Investigations should be fun and thought-provoking ways of leading students to the development of mathematical algorithms. Students who are actively involved in the discovery of mathematics will develop deeper conceptual understanding and stronger problem solving skills. Investigations should allow students to construct their own understanding of a task and make connections to their personal backgrounds. Investigations will also support student understanding of mathematical proofs. Most investigations are conducted in cooperative groups, thereby promoting cooperative behavior.

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<tr><th>Area of Concern</th><th colspan="2">Strategy</th></tr>
<tr><td rowspan="4">Word Problem Solving</td><td>Math journaling</td><td>It is imperative that students organize their work as well as their thinking. For many students, organizing their work will aid in the organization of their thoughts. In their math journals, students should document their thoughts about what worked and did not work during a task, including reasons for it. Students should write summaries of investigations and activities. They should also develop and maintain ongoing vocabulary sheets that record, define, and provide hints for words, terms, and symbols that might be new or unfamiliar. Students should be encouraged to use appropriate mathematical terminology once they become familiar with it. Journal prompts should be given regularly that require students to explain conjectures, processes, and conclusions. Leading questions should be provided that require students to express ideas and apply their learning to various situations and problems.</td></tr>
<tr><td>Splashes</td><td>Splashes can be used to assess students through their expression of ideas and concepts. Students are asked to summarize and explain the material and related topics from given concepts or chapters. Students should use a variety of representations to create splashes. Tables, graphs, symbols, and words can be used to represent the material, along with other creative representations designed by students. Splashes are a great way to assess students' understanding of the material, as well as their ability to explain, summarize, connect, and integrate topics.

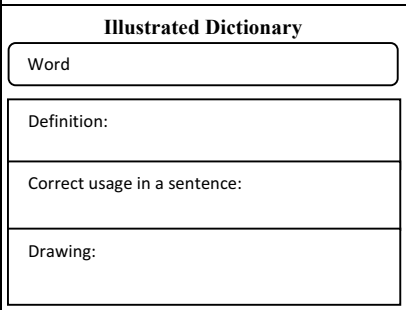
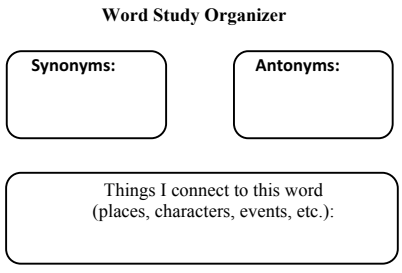
Splashes<table><tr><th>Diagram</th><th>Table</th><th>Graph</th><th>Rule</th><th>Connections</th></tr><tr><td>Draw a diagram to demonstrate understanding of information presented in the problem.</td><td>Display information in an organized manner using a table.</td><td>Graphs provide a visual display of information. Students can graph information even when graphs are not required as part of the problem.</td><td>The rule generalizes number pattern found in the problem.</td><td>Students write to explain connections between the following.
Diagram to table
Diagram to graph
Diagram to rule
Table to graph
Table to rule
Graph to rule</td></tr><tr><td colspan="5">Explain steps for deriving answer(s).
Answer problem's question(s) in one or more sentences.</td></tr></table></td></tr>
<tr><td>T-Chart</td><td>A t-chart is a two column organizational chart for showing work and explaining steps when deriving solutions.

<table><tr><td>Calculations</td><td>Explanation</td></tr><tr><td></td><td></td></tr></table></td></tr>
<tr><td>4-Step Method for Problem Solving</td><td>Step 1: Understand the Problem

This step involves more than just understanding the words in the question. You need to understand what the question is asking you to do. Points to consider when faced with a new problem include:

What is the question asking?
What type of math might be involved?
What is familiar about the problem?
What is different about the problem?</td></tr>
</table>

		Step 2: Recognize Important Information There are different kinds of information in a problem. Numbers are usually given in the question itself, in a figure, a graph, or a table. Mathematical information can also be given in the form of geometric figures, algebraic expressions, or word problems. Sometimes, you need to recognize information that is needed but not given. Another kind of information that is important is the prior knowledge that you bring to the question. This knowledge can also help you find a solution. Some questions to ask in Step 2 include: What do the numbers in the problem represent? What math skills or formulas are needed to solve the problem? What information is *not* needed? What additional information is needed to solve the problem? What steps could you take to find missing information? **Step 3: Select a Problem-Solving Strategy** After you understand what the question is asking you to do and have gathered the information you need, the next step is to make a plan for solving the problem. Points to consider when making your plan include: What math strategies will work for this problem? For multiple-choice questions, are the answer choices helpful? For open-ended questions, do the directions tell you how to solve the problem? What is your plan? **Step 4: Solve and Check** The final step includes both solving the problem and checking your solution. For this final step, you shoul Use the information gathered in the first two steps and the strategy selected in the third step. Write your work neatly and carefully, showing all steps. Read the question again to make sure you answered it correctly. For open-ended questions, check that you answered every part of the question. Check your calculations carefully or solve the problem using a different method. Make sure your answer seems reasonable.

Area of Concern	Strategy	
Vocabulary Development	**Identification and application of appropriate math vocabulary and symbols**	Using correct mathematical terms in journal writing, presentations, and class discussions is important. It is vital to teach students that being precise and accurate in mathematics includes the use of correct mathematical terms and symbols. Initial math discussions may include language familiar to students, but a shift toward incorporating appropriate mathematical terms should be implemented as soon as conceptual knowledge is acquired.
	Illustrated dictionaries	 Creating a compilation of vocabulary words with definitions, drawings, and correct usage in a sentence reinforces the meaning of the vocabulary studied. ELL students should be encouraged to create bilingual dictionaries to help with comprehension of specialized mathematics vocabulary.
	Library and Internet research	Library and Internet research provides students with opportunities to extend their understanding of a specific topic covered in class or explore the questions mathematicians are currently investigating. A study of mathematicians from the past and present can help students put a human face on the study of an abstract topic. Encourage students to investigate the first use of mathematical terms, symbols, and notation.
	Math Word Wall	**A word wall is:** π Created by students π **Visual** displays of concepts π A tool to use … **Guidelines:** π Add words gradually, as they appear in a lesson. π Make words very accessible by putting them where every student can see them. π Be selective about which words go on the wall; use terms that appear in the Core Curriculum, PSSA, and Keystone lists. π Make sure that Word Wall words are used correctly in classroom discussions.
	Word Study Organizer	 A word study organizer graphic organizer helps students extend their knowledge about a word or concept by forcing connections based on personal associations.
	Writing to explain math processes	This strategy emphasizes the importance of being able to articulate the what, how, and why of a math process in writing. A common tendency among students is to memorize steps to take to solve a problem without understanding the situation, the method used to solve the problem, and why that method is correct. Requiring students to explain the concept and process in writing will deepen their understanding beyond the simple memorization of steps. The ability to explain and retain the material will be strengthened by the writing process.

Math Areas of Concern

- Computational Fluency
- Fact Fluency
- Math Application: Concepts & Procedures
- Algebraic Concepts
- Word Problem Solving
- Vocabulary Development

Computational Fluency

The NCTM Principle and Standards of School Mathematics (2000) define computational fluency as having efficient, flexible and accurate methods for computing. The key thing to note is that it does not say compute using paper and pencil methods. Students need to be fluent in mental math, paper and pencil methods and using technology such as a calculator in computing answers to situations involving numbers (both whole numbers as well as fractions and decimals). In fact one often overlooked or underdeveloped aspect of computational fluency is not only being able to compute in all three ways but also knowing which method is best based on the given task. In addition, students must be able to determine if an exact answer or a close approximation (estimate) is sufficient. Another key idea is that just being able to compute the correct answer of estimate is not enough either. Students must be able to compute accurately in all three methods AND know when to do what operation. In other words that must be able to solve problems that involve numbers.

Fact Fluency

Math fact fluency is the ability to recall the answers to basic math facts automatically and without hesitation. Fact fluency is gained through significant practice, with mastery of basic math facts. It is very important that all students understand the concepts of addition, subtraction, multiplication and division. Students learn the commutative properties of both addition and multiplication.

Math Application: Conceptual & Procedural

Applying involves using one's conceptual and procedural knowledge to solve problems. A concept or procedure is not useful unless students recognize when and where to use it—as well as when and where it does not apply. In school, students are given specific problems to solve, but outside school they encounter situations in which part of the difficulty is figuring out exactly what the problem is. Therefore, students also need to be able to pose problems, devise solution strategies, and choose the most useful strategy for solving problems. They need to know how to picture quantities in their minds or draw them on paper, and they need to know how to distinguish what is known and relevant from what is unknown.

Algebraic Concepts

Algebra is an abstract concept in mathematics. Algebra is concerned with operations on sets of numbers or other elements that are often represented by symbols. Algebra is a generalization of arithmetic and gains much of its power from dealing symbolically with elements and operations (such as addition and multiplication) and relationships (such as equality) connecting the elements. Thus, $a+a=2a$ and $a+b=b+a$ no matter what numbers a and b represent. Students will be able to understand patterns, relations, and functions; represent and analyze mathematical situations and structures using algebraic symbols; use mathematical models to represent and understand quantitative relationships; and analyze change in various contexts.

Word Problem Solving

Word problems commonly include mathematical modeling questions, where data and information about a certain system is given and a student is required to develop a model. For example:

1. Jane has $5 and she uses $2 to buy something. How much does she have now?
2. If the water level in a cylinder of radius 2 m is rising at a rate of 3 m per second, what is the rate of increase of the volume of water?

Word problems are a common way to train and test understanding of underlying concepts within a descriptive problem, instead of solely testing the student's capability to perform algebraic manipulation or other "mechanical" skills.

Vocabulary Development

For new terms, descriptions, explanations, or examples are provided along with repeated reinforcement of terms through applications that involve multiple representations such as visuals, symbols, graphics and providing understanding in his/her own words. Misconceptions related to specific terms and where applicable, prefixes, suffixes and root words are addressed. The learner participates in activities that promote deeper knowledge of terms including written documentation in their personal notes and meaningful recreational drill and practice.

Resources

Gibson, Akimi and Judith Gold. *The Tutor*. LEARNS, 2002. http://www.pbs.org/launchingreaders/readingformeaning/helpfularticles_1.html

Horowitz, Sheldon H. *Reading Comprehension – Reading for Meaning.* New York, NY: National Center for Learning Disabilities, 2014.

International Reading Association. *Phonemic Awareness and the Teaching of Reading A Position Statement from the Board of Directors of the International Reading Association*. Newark, DE: International Reading Association, 1998. www.reading.org

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Marzano, Robert. *Building background knowledge for academic achievement: Research on what works in schools.* Alexandria, VA: ASCD, 2004.

National Institute of Child Health and Human Development. *Put reading first: Helping your child learn to read*. Washington, DC: U.S. Government Printing Office, 2001.

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National Council Teachers of Mathematics (NCTM) www.nctm.org